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SEARCH IN THE LABOR MARKET: INCOMPLETE CONTRACTS AND GROWTH

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Abstract

This paper shows that search in the labor market has important effects on accumulation decisions. In a labor market characterized by search, employment contracts are naturally incomplete and this creates a wedge between the rates of return and marginal products of both human and physical capital. As a result, when a worker invests more in his human capital, he increases the rate of return on physical capital. Provided that these factors are complements in the production function, this will increase the desired level of investment for firms. Then, because physical capital is not being paid its marginal product, the rate of return on all human capital goes up. Thus in this model there are *pecuniary increasing returns to scale* in human capital accumulation in the sense that the more human capital there is, the more profitable it is to accumulate human capital. Applying this argument conversely, the presence of pecuniary increasing returns in physical capital accumulation also follows. These pecuniary increasing returns lead to *amplified* inefficiencies and to the possibility of multiple equilibria. They also imply that factor distribution of income has an important impact on growth. Finally, the paper derives new links between unemployment and human capital accumulation and shows that when technology choice is endogenized, search introduces a negative wage formation externality which may lead to excessively fast diffusion of new technologies.

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1) Introduction

Progress of knowledge is undoubtedly the most important engine of growth. Yet, despite the fact that most of the new productive knowledge can quickly spread across countries, we still observe significantly different long-run growth performances. This leads many to believe that the incentives to acquire and apply this knowledge, therefore, the rewards to physical and human capital, differ across economies. Numerous factors ranging from social regard (e.g. Sawyer (1949), Baumol (1990), Cole et al (1991)) to coordination across sectors (e.g. Rosenstein-Rodan (1943), Murphy et al (1989)) are obviously important in this process. But economic historians also emphasize the role of institutions (e.g. Mokyr (1990), North (1981)) and in particular, the importance of the relations between labor and capital (e.g. Bean and Crafts (1993), Eichengreen (1993)) as determinants of long-run economic performance. Rewards to various skills and by implication the rate of return on physical capital are determined in the labor market. Therefore to understand how much of the available stock of knowledge will be exploited and extended by a society we need to study the organization of the labor market and the institutions governing trade *within* productive units. If trade necessary for productive relationships does not generate a high enough return to capital, sufficient investment will not be forthcoming. But neither will sufficient human capital be accumulated if various skills are not appropriately rewarded.

This paper starts from the premise that search is an important feature of most labor markets; both workers and firms have to engage in costly search activities when they are looking for a partner to produce. Our main argument is that the presence of search will create a wedge between the marginal product of labor and the wage rate (and also between the marginal product of physical capital and its rate of return) and that this will introduce important external effects in the process of human and physical capital accumulation. The role of human and physical capital externalities as a cause of divergent growth performance has been emphasized by Lucas (1988,1990) and by Romer (1986). Lucas assumes that when a worker increases his education, all other workers also experience increased productivity, hence there exists a *technological* externality in human capital accumulation. The seminal paper by Romer, on the other hand, assumes that technological social increasing returns are present in physical capital accumulation[1]. Social increasing returns as formulated by Lucas and Romer do not only suggest that the level of growth will be low compared to the first-best, but they also lead to the *amplification* of the inefficiencies: when an agent invests less, everyone else's output and productivity will be lower and they too will be induced

[1] Other strands of the literature build on Romer (1990) who relies on monopolistic competition and increasing returns at the firm level or on Rebelo (1991) who takes linear accumulation rules and constant returns to scale.

to invest less. In other words, the presence of this type of interactions imply that each agent's optimal level of investment is increasing in an economy wide average (or that there exist strategic complementarities in the sense of Cooper and John (1988)). However, a possible objection to the mechanisms put forward by these models is that it is not clear what underlies such technological externalities and also in many situations the importance of technological externalities *within a time period* seem limited (though "human capital externalities" across generations appear more plausible). This paper will show that even when such technological externalities are absent, search in the labor market will introduce *pecuniary social increasing returns to scale*[2] within a time period. That is, as workers invest more in their human capital, though they do not affect others' productivity, they increase *the rate of return* on other workers' human capital. Further, the same argument applies to firms' physical accumulation decisions and pecuniary increasing returns in physical capital accumulation will be present as well. Therefore the externalities we emphasize do not only lead to lower than optimal growth but they also imply similar equilibrium strategies to those of Lucas and Romer and they similarly lead to the *amplification* of the inefficiencies in the accumulation process.

The effects of the increasing returns, amplification and complementarities that we propose can be general, potentially influencing growth, development, locational choices, business cycles and even organizational forms[3]. However this paper is only a first attempt at suggesting that labor market related imperfections and the presence of two-sided investments will lead to this type of complementarities and will therefore try too illustrate them by means of a very simple model. Suppose that output is produced by a partnership of a worker and an entrepreneur and that both parties need to undertake some ex ante investment; the worker in human capital and the entrepreneur in physical capital. The efficient level of output will be produced if both parties are paid their marginal product and in practice there are two ways of ensuring this: (i) human and physical capital can be traded in a competitive (Walrasian) market; (ii) ex ante complete contracts can be written to determine the rewards to the different factors of production. However, when trade in the labor market is not regulated by the Walrasian auctioneer but requires bilateral

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[2] We suggest the term "pecuniary increasing returns" because the logical alternative, pecuniary externality, is often used when there is no market failure but only distributional effects. As in the case of pecuniary externalities, our effects do not originate from missing markets. However, because prices are not always equal to marginal cost in all markets, market mediated interactions still lead to increasing returns and significant inefficiencies.

[3] See the recent survey by Matsuyama (1994) on the wide range of potential applications of the complementarities (or of "circular and cumulative causation" as called by Mydral (1954)) introduced by firm level decreasing average costs and monopolistic competition.

search, both of these solutions run into problems. First, since search implies that most workers cannot costlessly move from one firm to another, wages and the rate of return on physical capital will be determined by bargaining on the quasi-rents created by this immobility. Second, search introduces anonymity; workers do not know who their future employers will be, as a result, they cannot contract with them and this leads to a natural *incompleteness of contracts*.

The implication is that wages and rates of return on capital will not be equal to the marginal products of these factors but instead will vary with the average product. Now the rest of the story comes together: as a worker invests more, he increases the average human capital that a firm expects to employ and because the return to both factors are positively related to average products, entrepreneurial profits will also increase. Next provided that human and physical capital are complements, an increase in the average human capital increases the average product of physical capital and firms find it more profitable to invest. Consequently, the average level of physical capital also increases and now by the same argument (i.e. that physical capital is not paid its marginal product), the rate of return on human capital goes up. Therefore by investing in human capital, the worker has improved the rate of return on physical capital and indirectly, the rate of return on the human capital investments of other workers (indirectly because this effect comes into operation only when firms respond to the initial investment). We thus end up with *increasing returns* but these are *pecuniary* not technological. The same argument naturally applies to physical capital investments, and pecuniary increasing returns in physical capital accumulation are also present. As a result the decision rules in our economy resemble those of Lucas (1988) and Romer (1986) and lead to amplified inefficiencies.

Obviously random matching (as assumed by most search models) and high costs of changing partners (in terms of foregone earnings in the process) are extreme assumptions and to the extent that our results depend on these, they will have limited applicability to more organized markets. To deal with these issues, we formally analyze wage determination under different matching assumptions and costs of changing partners. We show that very small search imperfections and ex ante anonymity of matching (rather than full randomness) are sufficient for all our results as long as they rule out Bertrand type competition. The important feature for our results is that decentralized trade should be subject to some transaction costs.

It is useful at this stage to relate our main mechanism to earlier literature. The search literature, most notably Diamond (1982) and Mortensen (1982) have stressed that "*actions taken now by one agent affect the probability distribution over future states that others experience*" (Mortensen (1982, p.968)). Although such externalities are theoretically appealing, their importance is limited. First there is a counter-

acting effect; as the market becomes more crowded some agents will also find it more difficult to find partners. Secondly, these externalities would manifest themselves in the form of increasing returns to scale in the matching technology and the evidence is that the matching technology exhibits constant returns (e.g. Pissarides (1986), Blanchard and Diamond (1989)) and thus no aggregate externalities seem to be present. This paper instead shows that important externalities are created by actions that affect the *value of future matches* and analyzes this in a general equilibrium setting. All our effects are derived from market mediated interactions (thus the term pecuniary) and do not come from the properties of the matching technology (as it would be the case in Diamond (1982) for instance).

Grout (1984) pointed out in a partial equilibrium setting that underinvestment would arise when investors could not capture the whole of the surplus they create because of ex post hold-up problems. Our first difference from Grout is that instead of assuming incomplete contracts and imposing severe liquidity constraints, all our results are derived from bilateral search which introduces the incompleteness of contracts and makes credit imperfections unnecessary. The more important innovation is the finding that two sided investment and search lead to increasing returns, to a natural amplification of the initial inefficiency and to the possibly to multiplicity of equilibria. Also the bilateral ex ante investment aspect and the ensuing pecuniary increasing returns imply that in contrast to Mortensen (1982) or Hosios (1989), no possible allocation of property rights over the surplus (and in contrast to Becker (1975)'s seminal analysis, no feasible contract) would restore efficiency. In this sense, our paper is related to the property rights literature (e.g. Grossman and Hart (1986), Hart and Moore (1989)) which analyzes in a partial equilibrium setting how incompleteness of contracts leads to underinvestment and the allocation of property rights can help by changing the division of rents. However, this literature does not obtain the economy-wide increasing returns we obtain from the general equilibrium interactions, nor does it derive the incompleteness of contracts from bilateral search. Davis (1993) and Caballero and Hammour (1993) respectively show how the effects emphasized by Grout (1984) will influence the composition of job qualities and the timing of job destruction and creation decisions. Finally Van Der Ploeg (1987) uses Grout's effect in a growth context to show the possibility of suboptimal growth rates. However, all of these papers have one sided investment. As a result, the pecuniary increasing returns, the amplification of inefficiencies (and the possibility of multiplicity) of our paper are absent in these models.

The basic model is described in section 2. The main results of the paper are contained in section 3. The rest of the paper extends our theoretical framework and obtains a number of new results. First, in section 3(ii), we show that there exists an optimal distribution of income across different factors of production. Second, we demonstrate in section 4 that multiple equilibria are possible. Third, we derive a

negative wage formation externality in the presence of technology choice and show how technology adoption may be excessively fast due to this externality. Finally in section 5, we endogenize unemployment, show how high unemployment discourages human capital accumulation and thus lead to a further multiplicity. Section 6 concludes. Appendix B contains the proofs of all the propositions while Appendix A offers a general equilibrium wage determination model that gives the wage rule we use in the main body of the paper as the unique equilibrium under different assumptions on heterogeneity and matching technology.

2) The Basic Model and The Competitive Allocation

We consider a model of non-overlapping generations. Each generation consists of a continuum of workers equal to 1 and a continuum of entrepreneurs also normalized to 1. The life of each agent consists of two parts. In the first, they choose their investment levels. Each worker's human capital decision involves the extent to which he wants to learn and extend the stock of knowledge that is already present in the economy (e.g. education). Similarly, entrepreneurs decide how much to invest in their skills. Production takes place in the second part of each agent's life in partnerships of one worker and one firm. Consumption takes place at the end of the period and then agents die[4]. The production function of a partnership is assumed to be constant return to scale and takes the form

$$y_t = A h_t^{\alpha} z_t^{1-\alpha} \tag{1}$$

where h_t is the human capital level of the worker and z_t is the skill level of the entrepreneur. The utility function of a worker of generation t is given as

$$v_w(c_t, l_t) = c_t - \frac{l_t^{1+\gamma}}{1+\gamma} H_{t-1} \tag{2}$$

where l_t is the human capital investment, γ is a positive parameter and H_{t-1} is the stock of human capital in the economy defined as[5]

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[4] Our economy is certainly very stylized but these assumptions are only adopted for simplification. The non-overlapping generations structure implies that entrepreneurs have to accumulate skills rather than physical capital but we will often think of z_t as physical capital. Also this setting requires that agents benefit from the stock of knowledge of their parents but again this is not a crucial assumption. If we use infinitely lived agents investing in human and physical capital in every period (e.g. as in Rebelo (1991)) and embed this in a search framework, we would still obtain the pecuniary increasing returns which are the key to all our results.

[5] This formulation will give us endogenous balanced growth. However, all the effects derived in this paper would also apply in an exogenous growth context where H_{t-1} grows exogenously rather than being determined by (3).

(3) $$H_{t-1}=\int_0^1 h_{t-1}^i di$$

where superscript i denotes worker i and will be dropped whenever this will cause no confusion. The human capital of the worker, h_t, is given by the following equation

(4) $$h_t=(1+l_t)(1-\delta)H_{t-1}$$

This formulation assumes that the worker absorbs and extends the stock of knowledge of either his parents or of the society by his human capital investment, l_t. Interpreting the utility function (2) in this way, we can argue that the cost of effort is proportional to H_{t-1} because the worker has to absorb this information (or alternatively higher human capital inherited from the earlier generations increases the value of leisure). According to (4), human capital depreciates at the rate δ if no further human capital investment is undertaken by the worker. The utility is maximized subject to (4) and the budget constraint

(5) $$c_t \leq W_t = w_t h_t$$

where W_t is the income level of the worker and w_t is the wage rate per unit of human capital. Each entrepreneur has a similar utility function given by

(6) $$v_e(c_t,e_t)=c_t-\frac{e_t^{1+\gamma}}{1+\gamma}Z_{t-1}$$

where e_{t-1} is the investment of the entrepreneur and Z_{t-1} is the stock of entrepreneurial skills of the economy at time t-1 and is defined similarly as

(7) $$Z_{t-1}=\int_0^1 z_{t-1}^i di$$

and also

(8) $$z_t=(1+e_t)(1-\delta)Z_{t-1}$$

which has a similar explanation to (4). Each entrepreneur maximizes her utility, (6), subject to (8) and the budget constraint,

(9) $$c_t \leq R_t = r_t z_t$$

where R_t is the total income of the entrepreneur and r_t is the return to entrepreneurial skill.

A complete description of behavior in this economy involves; (i) an investment decision for each worker and firm; (ii) given the distribution of types of workers and firms, an allocation of workers to firms and (iii) a wage level for each level of human capital and a rate of return for each level of entrepreneurial skill. The economy will be in *equilibrium* iff (a) given the distribution of types, their allocation and their payments are in equilibrium, and (b) given the final rewards, the ex ante investment decisions are privately optimal. We start with the Walrasian system which is frictionless and all allocations take place at one point in time. That is, the auctioneer calls out wage schedule as a function of human capital levels and rates of return on entrepreneurial capital, and trade stops when all markets clear. Note first that the allocation

problem with competitive markets is straightforward (e.g. Kremer (1993)); the most skilled worker will be allocated to the most productive entrepreneur because the marginal willingness to pay for the best worker is highest when the level of capital is highest. In other words, imagine a ranking of all entrepreneurs in descending order and then a similar ranking for workers, then worker i and entrepreneur j will be matched together when they have the same ranks in their respective orderings[6]. Given such an allocation, the competitive equilibrium wage rate and rate of return on entrepreneurial capital for each pair will be determined as follows:

$$w_t=w(h_t,z_t)=\alpha A h_t^{\alpha-1} z_t^{1-\alpha} \qquad (10)$$
$$r_t=r(h_t,z_t)=(1-\alpha)A h_t^{\alpha} z_t^{-\alpha}$$

Given these wage rates, the equilibrium accumulation decisions ("saving rates") will be given as;

$$l_t=\{(1-\delta)w(h_t,z_t)\}^{1/\gamma} \qquad (11)$$
$$e_t=\{(1-\delta)r(h_t,z_t)\}^{1/\gamma}$$

which set the marginal cost of investment equal to the marginal benefit. This gives us our first result which like all others in this paper is proved in the appendix.

Proposition 1: The competitive equilibrium is unique, symmetric[7] and Pareto optimal. All equilibrium paths converge to a unique balanced growth path along which the economy grows at the rate g^c where $g^c=(1+\{\alpha^{\alpha}(1-\alpha)^{1-\alpha}A(1-\delta)\}^{1/\gamma})(1-\delta)-1$ and has z/h ratio equal to $(1-\alpha)/\alpha$.

This economy intratemporaly fully efficient in the sense that more human capital investment by a worker will not increase the welfare of other workers and firms more than it reduces the pay-off to the worker. In particular, increasing all agents' investments by a small amount will not be Pareto improving. We therefore say that this model does *not* exhibit *social increasing returns*. This is similar to the growth model of Uzawa (1965) or Rebelo (1991) and is different from Romer (1986) or Lucas (1988). In Lucas' and Romer's models, an agent would improve welfare at a given point in time by investing more because of the external effects and moreover, each agent wants to invest more when others increase their investment

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[6] More formally define for every worker i, $\Omega_W(i)=\int_{s:\, h_s>h_i} ds$ and similarly for each entrepreneur j. Then i and j have the same rank iff $\Omega_W(i)=\Omega_F(j)$. In the competitive allocation (or efficient matching) i and j will be matched together iff (i) they have the same rank or (ii) when i is unmatched and has the lowest rank ($\Omega_W(.)$) among the unmatched workers and $\exists j^*$: $\Omega_W(i)=\Omega_F(j^*)$, and j has the lowest $\Omega_F(.)$ among remaining entrepreneurs.

[7] All workers choose the same investment level and likewise for all entrepreneurs.

because of the technological increasing returns to scale. These features are absent in this model because there are no technological externalities (within a period) and the labor market is competitive. Yet the current generation does not take into account the positive effect that it creates on the productivity of future generations by investing more. Nevertheless, the competitive allocation is still Pareto Optimal[8] because there is no way that the future generations can compensate the current one for the increased investment (i.e. each generation consumes in only one period and there is no asset that later generations can use to compensate earlier ones) and it is therefore not possible to increase investment and then by a series of redistributions make all agents better-off.

3) Search, Incompleteness of Contracts and Pecuniary Increasing Returns

We now turn to an economy in which productive partnerships are formed via anonymous random matching. At the beginning of the second period of their lives (i.e. after the investment decisions have been made), workers and entrepreneurs are matched one-to-one, so that no worker nor entrepreneur remains unemployed (see section 5, on this). However, despite this lack of unemployment, moving from one partner to another will be costly ("search"). Note that this economy has an explicit temporal dimension; investment decisions are taken first and allocations and equilibrium prices are determined later. This is obviously an extreme assumption; although important educational choices are taken before workers arrive to the labor market, an important part of the human capital is also accumulated on the job. Such an extension will not however change our main results[9].

Combined with the presence of mobility costs, the timing of investment choices will introduce a classic hold-up situation. Although how much each agent has invested may be contractible, they cannot write a contract with their partner to relate their payments to this investment level because the identity of this partner is unknown at the time of investment. Further, since changing partners at the search stage is costly, they have to share the surplus within the partnership and an agent who invests more has no way of making sure that he or she will receive the return of this higher investment. This implies that workers' and entrepreneurs' returns are in general related to the average product of their investment (as well as, or

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[8] This is obviously not the only Pareto Optimal outcome. Yet it is the only one in the set of Pareto optimal outcomes that can be achieved by a competitive price system and also the one that maximizes the welfare of the current generation. Throughout the paper we will use the term Pareto optimum to refer to this allocation.

[9] Acemoglu (1993b) shows that similar effects arise in a model of training if workers and firms cannot guarantee that their relationship will continue indefinitely. The main externality there is between the current employer of the worker and his future employers who also benefit from the worker's human capital.

instead of, the marginal product). For the main result of the paper we will assume that search imperfections lead to a wage determination rule whereby the worker obtains a proportion β of the total surplus, thus wages and rates of return on capital depend only on average product. Appendix A offers a wage determination model which gives this result as the unique equilibrium under a variety of assumptions on heterogeneity and matching technology. Although other assumptions on technology and bargaining would imply different sharing rules, all our qualitative results would go through whenever wages are different from marginal products irrespective of the magnitude of this wedge.

(i) Search in the Labor Market and Pecuniary Increasing Returns

In this section a worker who is in a match with total output y_i obtains $W_i=\beta y_i$ and matching is random. This implies that each worker has an equal probability of meeting each entrepreneur and likewise for each entrepreneur irrespective of their skill levels - naturally, in equilibrium some agents may decide to change partners so the equilibrium allocation will not be entirely random. From these assumptions it follows that the expected income of a worker with human capital h_t conditional on the average investment of entrepreneurs is given as

$$W(h_t,\int (z_t^i)^{1-\alpha}di)=\beta A h_t^{\alpha}\int (z_t^i)^{1-\alpha}di \tag{12}$$

The worker will always obtain a proportion β of the total output which will depend on the capital level of the entrepreneur z_t^i as well as his investment h_t. The expected income of an entrepreneur with capital z_t is similarly given by

$$R(\int (h_t^i)^{\alpha}di,z_t)=(1-\beta)A z_t^{1-\alpha}\int (h_t^i)^{\alpha}di \tag{13}$$

Total incomes are not directly related to marginal but only to average product. Consequently, the marginal reward to one more unit of investment is proportional to average product. This implies the following investment rules;

$$\begin{aligned} l_{t-1}&=\{\beta\alpha(1-\delta)A h_t^{\alpha-1}\int (z_t^i)^{1-\alpha}di\}^{1/\gamma} \\ e_{t-1}&=\{(1-\beta)(1-\alpha)(1-\delta)A z_t^{-\alpha}\int (h_t^i)^{\alpha}di\}^{1/\gamma} \end{aligned} \tag{14}$$

The difference between these expressions and (11) is worth noting. In both, pecuniary interactions are present. However, in (14) they take the form of *pecuniary increasing returns to scale*. By increasing its investment level a worker has a first order impact on the welfare of firms in contrast to (11) where the only impact was through the equilibrium rates of returns and thus was of second-order. Moreover, when a worker increases his human capital investment, this also has a first-order impact on the desired investment level of all the entrepreneurs. And by the same argument, the increase in the entrepreneurs' investment decision will have a first-order impact on the welfare and desired investment level of all workers. This is

the channel which introduces *pecuniary increasing returns* and *amplified inefficiencies* in the accumulation decisions; by investing less workers (firms) not only create a negative impact on the welfare of other workers and firms but also reduce their desired investment levels.

Proposition 2: The decentralized search economy with random matching has a unique equilibrium. All equilibria paths converge to a balanced growth path along which the economy grows at the rate g^d where $g^d=(1+\{\alpha^\alpha(1-\alpha)^{1-\alpha}\beta^\alpha(1-\beta)^{1-\alpha}(1-\delta)A\}^{1/\gamma})(1-\delta)-1$ and is less than g^c. The decentralized economy is Pareto dominated by the competitive equilibrium and exhibits pecuniary increasing returns in the sense that a small increase in all agents investment will make everyone better off.

Since part of the emphasis of this paper is on the amplification of inefficiencies due to pecuniary increasing returns, it is instructive to assess the quantitative difference between g^c and g^d. As we will see in the next subsection $\beta=\alpha$ minimizes the distortions in our economy. Therefore taking this as the base case would minimize the difference between g^d and g^c. Let us then take $\beta=\alpha$ and set these equal to the share of labor in total output, .6. We take $\gamma=1$ to simplify the expression. This implies that $g^d=0.51g^c-0.49\delta$. So if $\delta=0$, competitive growth rate is twice as high as our decentralized equilibrium. This gap widens when δ is positive. For instance with $\delta=.9$ which implies limited knowledge spill-overs across generations and A equal to 0.569, the decentralized economy grows at 2% in each period whereas g^c is equal to 13.5%, nearly seven times faster. These gaps become much larger when we take configurations where α is not equal to β (i.e. we do not have optimal distribution of income across factors). Because our model is very simple, this exercise should of course be interpreted with caution but it still suggests that the amplification of inefficiencies through our mechanism can indeed be quite significant.

(ii) The Role of Institutions, Property Rights and Optimal Factor Shares

The growth rate of the decentralized equilibrium, g^d, depends on β, the way that total output is shared among workers and entrepreneurs. The division of surplus between the factors will in general depend on the institutional structure of the economy and also on the allocation of property rights. Although investment and education subsidies financed by non-distorionary taxation could clearly restore efficiency, it will be more instructive to investigate whether efficiency can be achieved by changing institutions and property rights as captured by β (as in Mortensen (1982) and Hosios (1989)). However, in our model, due to the bilateral investment aspect, such a way of achieving efficiency is not possible and this again points out that the externality we propose cannot be easily removed.

Proposition 3: $\beta=\alpha$ achieves the highest balanced growth. For all values of β the economy is Pareto inefficient.

The intuition is straightforward, α captures how much total output increases by one more unit of human capital investment and $(1-\alpha)$ is the incremental contribution of entrepreneurial investment to output. To minimize distortions, workers should receive a proportion α of the total product they create and firms get a proportion $(1-\alpha)$. This also makes it clear that if only one of the parties had ex ante investments, first-best could be achieved by making that party the full residual claimant of the final returns. However in our case since both parties have important ex ante investments, they both need to be made full residual claimants and this is not possible. Expressed alternatively, efficiency can be restored if the share of the worker (and of the entrepreneur) can be conditioned upon their ex ante investments. Yet, this requires more than a straightforward reallocation of property rights. In particular, since it is not known who will form a pair, efficient provision of incentives requires multilateral contracts conditional upon ex ante investments. Although a social planner could implement a system of this sort, the decentralized equilibrium cannot easily achieve this. In summary, β which in our model represents the institutional arrangements in the labor market has important effects on accumulation decisions but no value of β restores efficiency in this economy[10].

4) Technology Choice, Multiple Growth Paths and Negative Wage Formation Externality

Section 3 demonstrated the presence of pecuniary increasing returns and amplification (or strategic complementarities). Such interactions can in general lead to a multiplicity of equilibria, yet given the set-up of the previous model, we obtained a unique equilibrium. In this section we will show that multiplicity of equilibria can easily arise in this setting. We will also show that in the presence of technology choice a negative wage formation externality is naturally introduced.

We modify our model such that the production function of a partnership is given as

(15) $$y_t=Ah_t$$

so output is linear in human capital. Entrepreneurs on the other hand have a technology choice which determines A. At zero cost they can choose a technology that has a marginal product A_1 or they can incur an additional cost kH_{t-1} and obtain the higher productivity A_2. The incremental cost of advanced technology

[10] Given the wage determination stage of Appendix A, more complicated sharing rules cannot arise in equilibrium. However, it is also straightforward to see that rules of the form $W_i=f(y_i)$ where f(.) is a nonlinear function but does not directly depend on ex ante investments will not help at all in achieving better allocations.

depends on the knowledge level of the society (H_{t-1}) because entrepreneurs have to absorb this knowledge too. Specifically this assumption will ensure the existence of balanced growth paths. Again in this section no worker or entrepreneur remains unemployed.

In this economy, there will in general be firms of high and low technology and as a result workers may want to switch from one firm to another. Appendix A shows that under some plausible assumptions, the possibility of such switches will not change the results of the previous section. However, when firms are constrained to choose between discrete alternatives, this may no longer be true. Thus in contrast to the previous section, the size of the mobility costs (the costs of changing partners) may become important. For this reason we start with high mobility costs which will illustrate the possibility of multiple equilibria; we will then turn to low mobility costs which will introduce the negative wage formation externality.

a) High Mobility Costs

High mobility costs imply that because changing partners is prohibitively costly, a worker will produce with the first firm he is matched with and wages will again be determined by bargaining between the entrepreneur and the worker without reference to "outside options". Therefore workers again obtain a proportion β of the total output, i.e. $W_i=\beta y_i$, as in the previous section. Now denoting the proportion of firms that adopt the high productivity technology at time t by τ_t, this implies that a worker of human capital h_t will have an expected wage equal to $\beta\{(1-\tau_t)A_1+\tau_t A_2\}h_t$. Since from (2), the worker is risk-neutral, the optimal human capital investment is given as

(16) $$l_t^m(\tau_t)=\{\beta((1-\tau_t)A_1+\tau_t A_2)(1-\delta)\}^{1/\gamma}$$

When all firms are expected to use the low productivity technology, the human capital investment of the workers will be given by

(17) $$l_t^m(0)=\{\beta A_1(1-\delta)\}^{1/\gamma}$$

In contrast when all firms are expected to adopt the technology with the higher marginal product, workers will optimally choose the higher level of human capital investment

(18) $$l_t^m(1)=\{\beta A_2(1-\delta)\}^{1/\gamma}$$

The profitability of the new investment for the entrepreneurs is obviously increasing in the level of the workers' human capital. In turn, workers are willing to invest more in human capital when all the firms possess the new technology because their average product is higher. Therefore as in the previous section, *pecuniary increasing returns* are present. By investing in the new technology, firms are making workers better-off and when workers invest more in response, all firms experience increased profits. Now defining, $B_1=(1-\delta)(1+\{\beta A_1(1-\delta)\}^{1/\gamma})$ and $B_2=(1-\delta)(1+\beta A_2(1-\delta)\}^{1/\gamma})$, we have;

Proposition 4: Suppose $W_i=\beta y_i$, then if $(1-\beta)A_1B_1 < (1-\beta)A_2B_1 - k$ and $(1-\beta)A_1B_2 < (1-\beta)A_2B_2 - k$, in every period there exist two pure strategy symmetric Nash equilibria, one in which the high cost technology is adopted and one in which it is not.

Intuitively, for the new technology to be productive, a large scale of production is required. In terms of our model, this corresponds to workers choosing a high level of human capital investment. However, workers will only do this when they expect high rewards (i.e. high average products), and this will be the case when all their future partners are expected to choose the more productive technology. In the previous section although the same forces were present, the Cobb-Douglas functional form led to a unique equilibrium. Here due to the discreetness of the technology choice, multiple equilibria can easily arise[11].

b) Small Mobility Costs

With high mobility costs, the advanced technology creates a positive externality on workers and indirectly, a positive externality on fellow entrepreneurs. When the mobility cost is small and there is the possibility of heterogeneity across firms, it may no longer be appropriate to assume that $W_i=\beta y_i$ for all pairs. The intuition is clear, when a worker is matched with a firm of low technology and changing partners is cheap, he would want to move unless he is paid more than a proportion β of the output of the low productivity firm.

Let us denote the cost of changing partners by ϵ as in Appendix A. This implies that the worker knows that he will find a new partner if he incurs the mobility cost ϵ. In particular again let τ be the proportion of entrepreneurs who adopted the more advanced technology. If the worker keeps moving until matched with a more productive entrepreneur his expected cost will be[12] ϵ/τ. Therefore the wage rate that

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[11] To see the importance of the discreetness note that in the low equilibrium an entrepreneur would prefer to increase its productivity slightly above A_1 if she could do this at a small cost. If possible, such actions would unravel the multiplicity of equilibria. It is therefore the non-convexity (discreetness) that forces her to choose between A_1 and A_2 and thus leads to the multiplicity. The negative externality of the next subsection is likewise due to the non-convexity.

[12] This is not strictly correct in general since we must use the conditional probability that an unmatched entrepreneur has the more productive technology and this may be different from the proportion of entrepreneurs who have chosen the more productive technology. But we will only look at the cases where τ tends to zero and to 1, and in both cases (19) applies exactly. Also note that this is a special case of Lemma 3 in Appendix A. All workers will have the same human capital level, h, thus $\Omega_W(i)=0$ for all i. Therefore they have exactly the same rank as the high technology firms and they should all receive $\beta A_2 h$.

a worker with human capital h receives when matched with a low productivity firm has to be

(19) $$w_1=\max\ \{\beta A_1 h,\ \beta A_2 h-\frac{\epsilon}{\tau}\}$$

for him not to switch. This equation is straightforward to explain. The outside option of the worker is what he can get in a more productive firm minus the cost of moving to such a firm. If this is less than his share of the pie he will obtain his share and if it is more, his outside option would be binding and he would obtain his outside option (either he gets this amount in his initial partnership or he switches and obtains this in expected value). In the previous sections, the outside option was never binding whereas it is now.

From equation (19), we can see that as a firm invests, the outside of option of all workers will increase and all firms using the old technology have to pay higher prices. This looks very much like a pecuniary externality that is also present in competitive models; as the demand for labor increases, the wage rate goes up. However, a pecuniary externality in a competitive labor market will increase the wage rate that is faced by all firms thus will not *bias* the technology choice. In our case the outside option is never binding for firms that have adopted the advanced technology and wages are set with reference to average product. Therefore as workers increase their investments, high productivity entrepreneurs make more profits. Yet low technology firms are forced to pay higher wages because the outside option of their workers becomes binding. Consequently, as more firms invest in the new technology, a *negative wage formation externality* is created and the low technology becomes less attractive relative to the advanced technology.

To analyze the interaction in this case more carefully, let $\epsilon \to 0$. If no firm chooses the advanced technology, for all positive values of ϵ, workers will have no binding outside option and will obtain βAh. In contrast, when a high number of firms choose the more advanced technology, from (19) workers can always obtain $\beta A_2 h$ (either they meet or switch to a high productivity firm or their outside option is binding when they bargain with a low productivity firm). Anticipating this, they choose their human capital investment equal to $l_{t-1}(1)$ as in (18) above. At this level of human capital investment, return to low technology is proportional to $\{A_1B_2 - \beta A_2B_2\}$. Whereas the return to choosing the more productive technology will be proportional to $\{(1-\beta)A_2B_2 - k\}$.

Proposition 5: As $\epsilon \to 0$, if $(1-\beta)A_1B_1 > (1-\beta)A_2B_1 - k$ and $A_2B_2 - k > A_1B_2$ there exist two symmetric pure strategy equilibria, one in which the high productivity technology is adopted and one in which the low productivity technology is used. The advanced technology can be adopted even when it is not socially optimal to do so.

As a result of the negative wage formation externality, the economy may experience too fast diffusion of new technologies. When a high number of firms are expected to adopt this technology, firms anticipate that they will have to pay higher wages even when they do not possess the new technology and the higher human capital of workers makes the new technology sufficiently attractive. However, it has to be borne in mind that the negative externality counteracts the positive externality we have emphasized so far, thus it may reduce the inefficiencies. But, since it can more than offset the positive externality, excessively fast technological diffusion may also result[13].

5) Unemployment, Human Capital Accumulation and Multiple Equilibria

In this section we relax the assumption that all workers find employment. This will introduce a number of new effects and a new source of strategic interactions. In order to illustrate these points we maintain the model of the previous section but instead of a technology choice entrepreneurs have to decide whether to be active or not. In the previous section both sides had ex ante investments and here workers have a human capital investment and firms have to decide ex ante whether to be active or not; this is also an "ex ante investment" sufficient to give us all our results.

The utility of entrepreneurs is now given by

$$v_e(c_t,e_t)=c_t-\mu(E_t)e_tH_{t-1} \tag{20}$$

where e_t only takes the values 0 or 1. The former denotes the entrepreneur deciding to be inactive whereas $e_t=1$ implies that the entrepreneur has decided to allocate her talents to production. E_t is the measure of active entrepreneurs. The cost of becoming productive depends upon H_{t-1} because in the same way as the worker has to absorb the stock of knowledge of the society so does the entrepreneur. It also depends upon E_t, the number of agents who decide to become entrepreneurs at time t because of decreasing returns when the entrepreneurial market becomes congested[14]. A formulation that captures the idea that the costs to entrepreneurship are related to search is to derive $\mu(E)$ from the standard matching function (e.g. Pissarides (1990)). According to this interpretation, each entrepreneur incurs a set-up cost η_0 and also decides how

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[13] Young (1992) for instance argues that such excessively fast adoption occurred in Singapore but attributes it to government policies.

[14] This type of congestion would follow from search, product or factor market competition. If this type of decreasing returns were not present, we would have two extreme equilibria, one in which all entrepreneurs are active and one where none are. Also as noted before, the mobility cost ϵ we have used so far can be derived from a standard matching function by relating it to time lost while searching for a new partner (which we would require the second part of each agent's life to be specified as a continuum) and this already implicitly assumed the possibility of unemployment but we preferred not to concentrate on it up to this point.

many vacancies to open. When there are more open vacancies, the probability that each vacancy will get filled falls and each entrepreneur needs to open more vacancies. Note in particular that according to this interpretation, the matching technology can be constant returns to scale as found by Blanchard and Diamond (1989); thus in no point we will require increasing returns in the search technology. The crucial ingredient is the natural one that the probability of a vacancy getting filled is decreasing in the number of vacancies. To capture all these features we can assume that the search technology takes a Cobb-Douglas form where the number of jobs (or active employers) is given as $E=V^{\nu}U^{\xi}$ where U is the number of workers looking for a job (which is equal to 1 in our model) and V is the number of vacancies. The key assumption for us is that $\nu<1$ (i.e. $\nu+\xi$ can be less than or equal to 1). This implies;

$$\mu(0)=\eta_0 \qquad \lim_{E\to 1}\mu(E)=\infty \tag{21}$$

The intuition is simple. As the number of vacancies goes to zero, the probability that a vacancy becomes filled is 1, thus an entrepreneur can become active at no additional cost except for the set-up expense η_0. When all entrepreneurs are active, each entrepreneur needs to open a very high number of vacancies in order to be matched with a worker. Since in general not all entrepreneurs enter the market, there will be unemployment among workers. In particular when E entrepreneurs are active, the unemployment (rate) will be equal to $u_t=1-E_t$.

Random matching implies that each worker has a probability $(1-u_t)$ or E_t of ending up with an entrepreneur. Since we assumed that the worker's human capital is not productive when he is unmatched, his expected return is

$$\beta(1-u_t)Ah_t \tag{22}$$

Therefore utility maximization implies

$$l_t^r=\{(\beta(1-u_t)A(1-\delta)\}^{1/\gamma} \tag{23}$$

The investment decisions of workers and the growth rate of this economy will be further distorted due to the presence of unemployment in (23); in particular, the higher is the unemployment rate of the economy, the lower will the growth rate be. However this is only a partial equilibrium result since unemployment is also endogenous, so the correlation between unemployment and growth will depend on how structural variables differ across economies and time periods (see Aghion and Howitt (1992), Bean and Pissarides (1992), Acemoglu (1993b)).

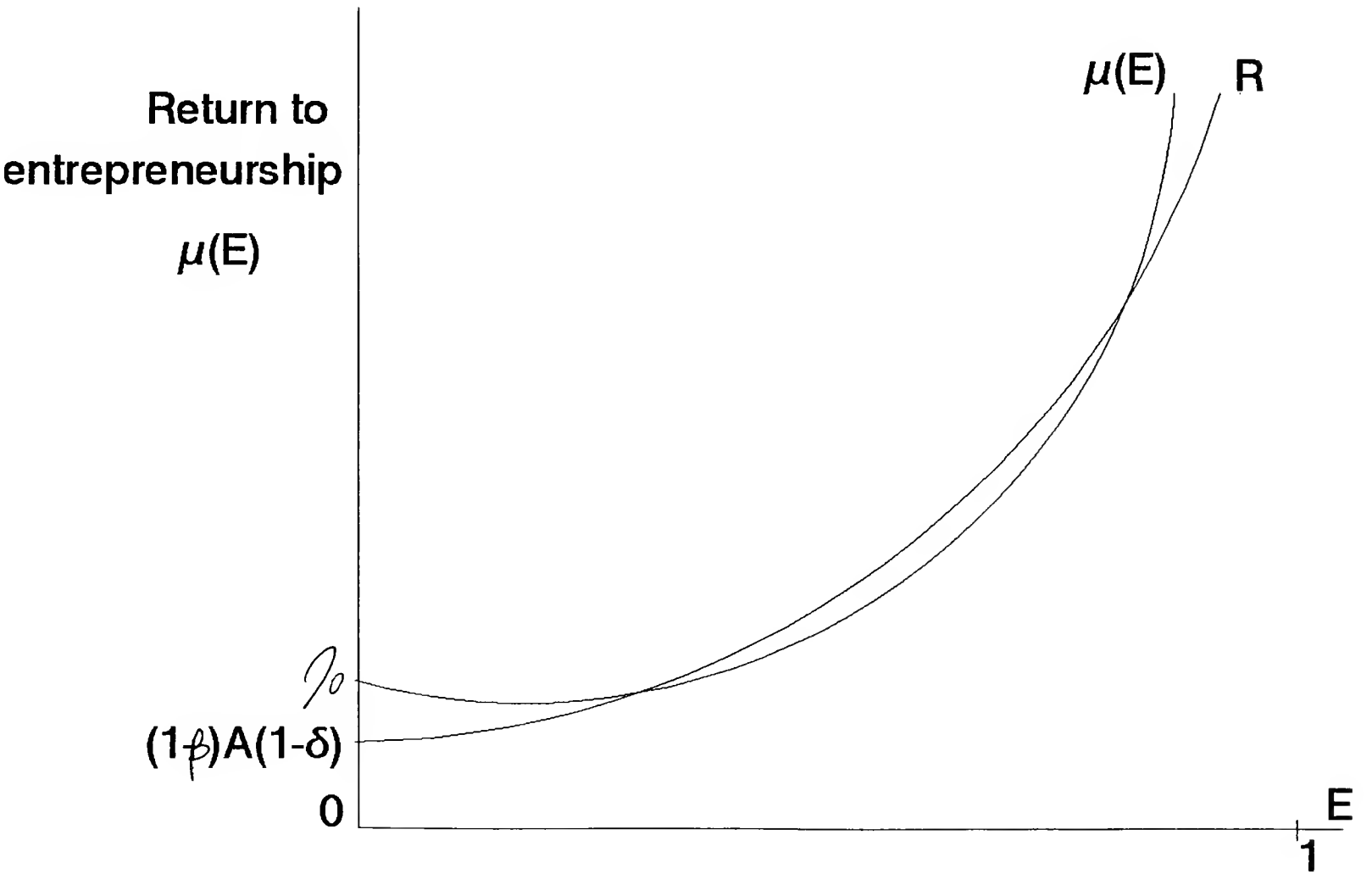
Next we need to determine the unemployment rate in this economy. For this we turn to the behavior of the entrepreneurs. Entry will stop only when return to entrepreneurship is equal to its cost, thus equilibrium requires

(24) $$\mu(E_t)=(1-\beta)A(1-\delta)(1+\{\beta E_t A(1-\delta)\}^{1/\gamma})$$

Inspection of (23) and (24) indicates the presence of *pecuniary increasing returns to scale*. When an entrepreneur decides to enter the market, all workers are made better off because the probability that they will be unemployed falls. And in response to this they decide to invest more - equation (23). When workers invest more, all entrepreneurs are made better off because they share the increased productivity of the workers. This amplification of the initial inefficiencies also leads to the possibility of a further multiplicity of equilibria as Figure 1 shows. The two curves that denote the left and right-hand sides of (24) are both increasing in E_t. These curves can intersect more than once and equilibria with different unemployment and growth rates are possible. The intuition of this multiplicity shows some similarity to that of Pissarides (1992) where when the proportion of the long-term unemployed in unemployment is high, fewer vacancies open and thus the level of unemployment remains high. In our economy, the effect does not work through the loss of skill of the unemployed but via forward looking choices of the workers on how much human capital investment to undertake. When unemployment is low, workers find it profitable to invest more in human capital but since labor is not paid its marginal product, entrepreneurs benefit from this higher investment and are more willing to be active which then reduces unemployment. The result is also similar to Diamond (1982) and Mortensen (1991) where *technological increasing returns* (respectively in the matching technology and the production function) lead to multiplicity of equilibria. However, our result does not depend on such technological assumptions. Robinson (1993) is another paper that has a similar result, especially as it relates human capital formation to unemployment as we do here, but again based it is based on technological externalities. Finally, Acemoglu (1993b) also obtains a complementary result that high unemployment discourages technological innovation and human capital accumulation but the reason is that with high unemployment, the effective cost of training that firms take into account when adopting new technologies is higher.

At this stage it is instructive to also look at the efficient outcome of this economy. For this purpose we define the constrained Pareto efficient allocation where the social planner is constrained by exactly the same matching imperfections as the decentralized economy. Since the social planner cannot make sure that only workers who will get jobs should be educated (i.e. random matching), all workers will have to invest in their human capital and some of them, just as in the decentralized outcome, will be unemployed and not use their skills. The crucial difference is that the social planner will internalize the spillovers and instead of setting the ex ante investment given by (23), he would choose $l^*(E)=\{A(1-\delta)E\}^{1/\gamma}$, thus in investing in human capital, he will take into account that entrepreneurs will also benefit from this investment level. Similarly, in (24) entrepreneurs ignore the positive externalities they create on the workers, neither do they

FIGURE 1



consider the negative impact they have on fellow entrepreneurs by increasing their entry costs. The social planner will internalize all these effects and replace (24) by

(25) $$\mu(E)+E\mu'(E)=A(1-\delta)(1+\{A(1-\delta)E\}^{1/\gamma})$$

Proposition 6: Consider the economy described above. If $\eta_0>(1-\beta)A(1-\delta)$, there exist either one equilibrium with no activity or three balanced growth path equilibria with different unemployment rates[15]. Pecuniary increasing returns are present. Equilibrium with lower unemployment Pareto dominates the ones with higher unemployment and all equilibria are constrained Pareto inefficient.

6) Concluding Remarks

The main argument of this paper is that imperfections in the labor market have important effects on accumulation decisions. Search and related imperfections create a wedge between the marginal product of factors of production and their rates of return, as a result they distort the investment incentives for accumulable factors. Also because of search, a natural incompleteness of contracts is induced and this prevents the possibility of providing the right incentives at the ex ante investment stage. However this is only tip of the iceberg; when workers accumulate less human capital, this not only depresses the growth rate but, since the wage rate is not equal to the marginal product of labor, it also reduces the profitability of physical capital investment. As a result, there will be less physical investment but, by the same argument, this reduces the rate of return on human capital and all the workers will also want to invest less. Consequently the original inefficiency is amplified and increasing returns are obtained, yet the externalities are not technological but derived from interactions in the labor market.

The paper shows that this type of framework implies that labor market institutions have a direct impact on the rate of growth and that there exists an optimal distribution of income across factors. Further, a number of other interesting results naturally follow. First, a multiplicity of equilibria is shown to exist. Second, the possibility of a negative wage formation externality with small mobility costs is illustrated. Finally, when unemployment is endogenized, a new link between human capital accumulation and unemployment and a new source of multiplicity of equilibria is obtained.

This paper is a first attempt at pointing out important interactions between search and

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[15] Note that because of our simple setting, the equilibrium at time t does not affect the likelihood of high or low unemployment equilibrium at time t+1. Therefore, there exist non-balanced growth equilibria in which there is high growth and low unemployment for a number of periods and low growth and high unemployment for some other periods.

accumulation/investment decisions. Many of the effects suggested in this paper require more research. First, equilibrium wage determination with matching and heterogenous agents needs to be fully solved (for an attempt at this see Shimer and Smith (1994)). Although the effects emphasized in this paper will survive there, new effects may also be introduced. For instance, interesting questions to be answered are; whether with sufficient heterogeneity, small mobility costs create a "large" divergence between marginal products and rates of return under different modelling assumptions; what the extent of equilibrium misallocation of workers to firms will be in the presence of non-negligible mobility costs; what the effects of efficient matching are in the presence of unemployment (on this see Acemoglu (1994)). Second, a more forward looking model of capital accumulation would be useful. Using such a model, the quantitative effects of the mechanisms proposed in this paper can be evaluated more carefully. Third, an investigation of the empirical relationship between factor shares and growth rate of output and of productivity will be informative.

Finally and most importantly, it should be investigated whether the economy can find ways of dealing with the imperfections created by the process of costly exchange. The first candidate is to change the organization of the markets. We know from the accounts of economic historians that the process of industrial development went hand in hand with the development of markets (for a fascinating account see Braudel (1982)) and that in general a number of markets have become more centralized in the process while also certain others such as the labor market can be argued to have become more "decentralized". It will be interesting to investigate what the conditions for centralization and for this process to be socially efficient are and how they interact with the process of (human) capital accumulation. The second candidate is to change the organizational forms of firms. An example of such a phenomenon is analyzed in Acemoglu (1993a) where changes in the internal organization of the firm correct "general equilibrium" inefficiencies created by moral hazard and adverse selection. No such solution is possible in the model of our paper, but with more structure, the joint determination of organizational forms and market allocations can be studied. I hope to be able to pursue these issues in future work.

Appendix A - Wage Determination

(i) A Simple Model of General Equilibrium Wage Determination

Let us assume that once a pair is formed, both parties incur a cost equal to ϵ when they change partners. This can be interpreted as a monetary or non-monetary mobility cost or a flow loss because finding a new partner takes time[16]. We will show that in our framework, even for very small values of ϵ, the rates of return will be completely decoupled from the relevant marginal products. We also assume that no agent can simultaneously bargain with more than one party and Bertrand type competition is ruled out. Thus our bargaining stage will be similar to Shaked and Sutton (1984) which is an extension of Rubinstein's (1982) framework. More precisely we assume the following structure. After pairs are formed following the random matching stage, the entrepreneur (firm, F) and the worker, W, play a subgame $\Gamma(y)$ (Figure A1) where y is the total output produced if there is agreement in this subgame. First, the firm makes a wage demand (node A) which can be refused by the worker (node B). If the worker refuses, he can at no cost make a counter offer (node C) or decide to leave and find a new partner at cost ϵ (to both parties). If he makes an offer, the entrepreneur can refuse this (node D) and quit at cost ϵ. If she decides to continue with bargaining, Nature decides whether the firm or the worker will make the last offer (node E)[17] and at this stage quitting is no longer permitted. The probability that the worker will make the offer is denoted by β. This structure in a simple way captures the importance of institutional regulations that determine the balance of power between workers and firms. If β is high, the worker has a strong bargaining position and vice versa. We choose β as a parameter rather than use an infinite horizon game with alternating offers and equal discount factors because this formulation enables us to investigate later whether a particular value of β (which can be also seen as a particular allocation of property rights on final returns) would internalize the externalities we emphasize. In the case where one of the parties terminates bargaining, both find new partners (incurring the cost ϵ) and play a similar subgame $\Gamma(y')$ (where y' is not necessarily the same as y if there is heterogeneity). The presence of the mobility cost ϵ is the crucial feature for us; it captures in our setting that search and changing partners are costly. Without this cost our economy would be frictionless. As ϵ becomes smaller, the amount of frictions are diminished; in the standard matching framework where search costs are foregone earnings, this corresponds to the discount rate approaching 1.

At any point in time, this economy will have a complicated history which describes the way agents have matched and bargained up to that point. We are only interested in equilibria that do *not* depend on this history in a complicated way or are Markovian and implies that all agents will play the same strategies

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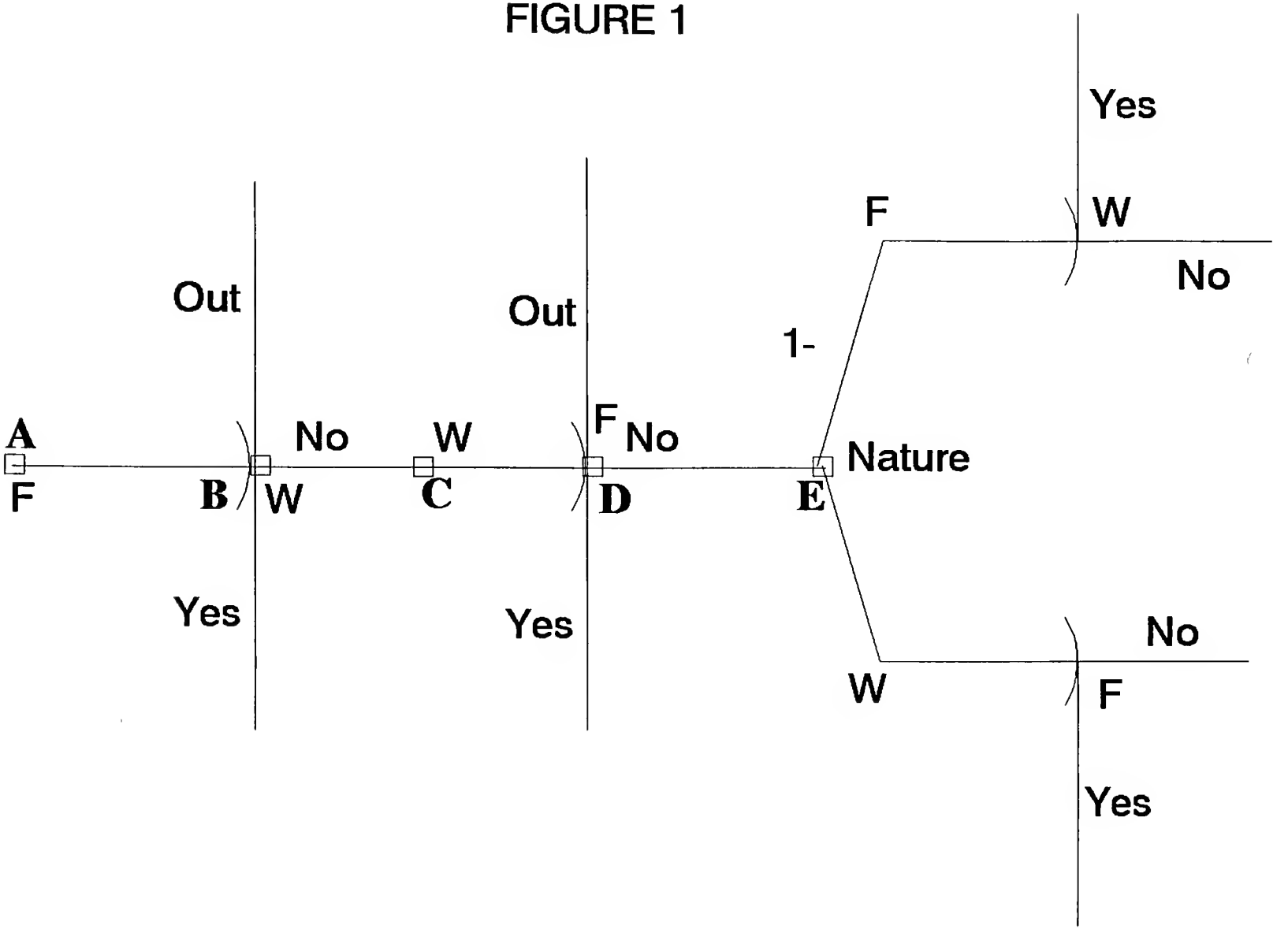
[16] With a finite number of agents, when a worker (or an entrepreneur) decides to end bargaining, no new partners may exist. This would imply that switching partners would be even less attractive thus bias the result in our favor. Since we have a continuum of agents, we can avoid this problem by assuming that there always exists some unmatched agents so that switching partners is always possible, yet the set of unmatched agents may be of measure zero. This modelling assumption is also a convenient substitute for the more rigorous strategy of looking at the steady state of a bargaining market with entry and exit (e.g. Osborne and Rubinstein (1990)).

[17] We have chosen Nature to move at node E to simplify the game tree, however an alternative game form where the worker and the entrepreneur asymmetrically alternate in making offers would do equally well as long as there is some sort of discounting (e.g. a small probability that the game will end without agreement after a rejection). However, applying the Nash solution irrespective of the extensive form would not be satisfactory since this may not correspond to the equilibrium of a well specified bargaining game (see Binmore, Rubinstein and Wolinsky (1986)). It can also be argued that in Rubinstein's (1982) framework with alternating offers, β will only depend on discount factors. But, in real world bargaining situations many other features seem to be more important in determining the relative bargaining strength.

FIGURE 1



in each completely identical subgame $\Gamma(y)$. We would also want the equilibrium to be subgame perfect, thus we are only interested in Markov Perfect Equilibria (see for instance Fudenberg and Tirole (1991)).

Lemma 1: With homogenous agents, random matching and the subgame $\Gamma(y)$ as described above, $W_i=\beta y$ is the unique equilibrium for all strictly positive values of ϵ.

Proof: Consider the extensive form in Figure A1 and let us denote the supremum expected return of the entrepreneur by V^S_E which we will try to determine. If the game reaches node E, the expected return of the worker is ßy and that of the firm is (1-ß)y. Next consider node D; since all pairs are playing the exact same game, the maximum the entrepreneur can expect from changing partners is $V^S_E-\epsilon$. On the other hand, she can say no and stay in, reach node E and obtain (1-ß)y. Since we are looking for the supremum of her equilibrium pay-offs, let her return at D be given by the maximum of these two amounts. Therefore at node C the worker has to offer the firm $\max\{V^S_E-\epsilon,(1-ß)y\}$. Now move to node B, the worker can say no and continue with bargaining in which case the game will proceed to node C and he will obtain $y-\max\{V^S_E-\epsilon,(1-ß)y\}$. Alternatively, he can say no and leave to obtain his outside option. Since we are after the supremum of the pay-off set of the firm, let us suppose that the worker gets his infimum, denoted by V^I_W. It then follows that at node A, the supremum of the pay-off set of the entrepreneur is

(A1) $$V_E^S=y-\max\{V_W^I-\epsilon,y-\max\{V_E^S-\epsilon,(1-\beta)y\}\}$$

Now noting that $V^S_E+V^I_W=y$, this equation has a unique solution which is $V^S_E=(1-ß)y$.

We can now repeat the above argument with the infimum of the pay-off set which will imply that $V^I_E=(1-ß)y$. Thus given the extensive form, there is a unique equilibrium irrespective of the value of ϵ (as long as it is not equal to zero) in which the worker receives ßy. QED

This lemma tells us that even with very small search frictions, there may be a large wedge between the marginal product of factors of production and their rates of return. The intuition is simple. If the worker could ensure that when he left the firm, he would get his marginal product and the entrepreneur likewise would get her marginal product, the pair would just bargain over the surplus, ϵ. However, in equilibrium, when the worker leaves, he will meet another firm and enter a very similar bargaining situation. In fact, if the firm he meets is exactly the same as the one he is bargaining with, he cannot expect anything better and hence his outside option will not be binding (i.e. the threat to quit is not credible) and bargaining will take place over the whole of the pie and the "inside" option will determine wages ("inside" option is defined as the return from $\Gamma(y)$ without the right to exercise the outside option). This intuition is very similar to Diamond (1971)'s famous result and to Shaked and Sutton (1984). It follows from this results that if we augment the model of the paper with the wage determination game outlined here, the result of Proposition would be the unique symmetric equilibrium. The analysis of the next subsection shows that in some other cases we can actually also rule out non-symmetric equilibria.

The limitations of this result should also be noted. First, it does not always hold in the presence of heterogenous agents because the worker (or the entrepreneur) may be moving to a better partner. We will return to this point in the next subsection where we will show that with efficient matching, the presence of heterogenous agents does not change our results and with random matching, we again obtain exactly the same results if ϵ becomes arbitrarily small. Second, we are not allowing Bertrand type competition so that a worker is never able to bargain with two firms simultaneously. If we allow a firm to meet two workers (or vice-versa) with a certain probability, then there will be a closer relationship between the rates of return and the marginal products but the competitive outcome will not be achieved unless we remove the mobility costs completely. It is natural that in our context that two workers would never simultaneously meet an entrepreneur and vice versa. If a worker is bargaining simultaneously with

two firms they will both get zero surplus. Thus ex ante a firm will have no incentive to contact a worker who is already in negotiation with, or employed by, another firm. Third, although I believe this formulation, where each party has the right to exercise the outside option, captures the most important and realistic effects, we know from the literature surveyed and analyzed by Osborne and Rubinstein (1990) that when Nature decides when bilateral bargaining will end and parties will be forced to take their outside options, bargaining markets converge to the competitive equilibrium as the discount factor tends to 1, e.g. Gale (1987)[18].

(ii) Robustness of Wage Determination: Heterogeneity and Efficient Matching

The divergence between marginal products and rates of return plays a crucial part in our results. Three assumptions have been used to obtain this result. The first is random matching; the second is to concentrate on symmetric equilibria (i.e. homogenous agents in Lemma 1) and the third is that multilateral contracts cannot be written before the investment stage. Relaxing the last assumption would take us away from the investigation of the decentralized equilibrium[19] (but see concluding remarks). In this section we will relax the first two.

Agents certainly do not randomly run into each other when they want to trade. Prices and the distribution of characteristics influence who will trade with whom. Is random matching the source of our results? To investigate this we define the polar case; efficient matching. If the matching technology is *efficient*, initial matches are organized so that the highest skilled worker is matched to the entrepreneur with the highest capital. In other words, the initial match will be the same as the competitive allocation analyzed in section 2; for instance i and j will put together in the initial match if they have the same rank (see footnote 6). This implies that in the presence of efficient matching, more investment increases not only the average product but also the likelihood of ending up with a good job. Such a matching technology may in general take us nearer to a competitive allocation where wages not only distribute the final returns across different factors of production but also allocate the "right" workers to the "right" jobs.

Lemma 2: Let us assume that matching is efficient and agents are potentially heterogenous at the wage determination stage. Denote the total output of worker i with entrepreneur j by y_{ij}. Then in the unique equilibrium we have $W_i=\beta y_{ij}$ where entrepreneur j and worker i have exactly the same ranks.

Proof: Consider worker i and entrepreneur j such that $\Omega_W(i)=0$ and $\Omega_F(j)=0$ (i.e. highest skilled entrepreneurs and workers). We first want to establish that $W_{ij}=\beta y_{ij}$ for this pair. Suppose one of the parties switches, they will move to subgame $\Gamma(y)$ where $y \leq y_{ij}$. Suppose for concreteness that the worker

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[18] The closest structure to our paper is in Bester (1989) which has a spatial competition model where moving between locations is costly. He shows that as this cost (which corresponds to our mobility cost, ϵ) tends to zero and discounting disappears, the equilibrium converges to the competitive outcome. However, this is due to the assumption that at the beginning of each subgame Nature decides who will make the first offer and there is discounting within the bargaining subgame. Thus with the cost of moving going to zero, any buyer who is not the first to make the offer can quit and eventually find a match where he will be the first one to do so and vice versa for the seller. This implies that with the mobility cost arbitrarily close to zero, the outside option cannot be worse than the inside option which can only be true in the competitive equilibrium. In our case, after a separation, agents find themselves at an identical node and thus even for very small values of the mobility cost, the inside option may be preferred to the outside one.

[19] Also this would require the relaxation of the ex ante anonymity assumption but Rubinstein and Wolinsky (1990) show that when this assumption is relaxed there exists no limit theorems for the decentralized equilibrium converging to the competitive outcome.

meets a firm j* with $z_{j*} < z_j$ (if this is an equality the argument of Lemma 1 applies). Obviously, the worker can only get more than W_{ij} if the entrepreneur gets below her inside option when matched with the worker of the initial match, βy_{i*j*}. But $y_{ij*} < y_{ij}$ also implies that the loss that entrepreneur j* is making must be larger than the gain of worker i. Thus if worker i wanted to switch, she would too. Therefore in equilibrium, worker i cannot get a higher wage in any subgame $\Gamma(y)$ and hence would end-up strictly worse off for all positive values of ϵ. The same argument applies to entrepreneur j. Now given that the highest ranked workers and entrepreneurs choose not to switch, the same argument applies to the next highest ranked and they too can only move to a worse match, so the outside option is not binding, etc. QED

Thus this lemma demonstrates that the divergence between rates of returns and the marginal products is not due to our random matching assumption but caused by the presence of transaction costs of decentralized trading (costs of breaking-up partnerships or finding new partners). Next we can also state a similar result for random matching with heterogenous agents but only when ϵ is arbitrarily small. Yet we have been unable to characterize the full equilibrium when ϵ is non-negligible, agents are heterogenous at the wage determination stage and matching is random;

Lemma 3: Let us assume that matching is random and agents are potentially heterogenous at the wage determination stage. Denote y_{ij} as the total output of worker i with entrepreneur j. Then as $\epsilon \to 0$, the unique equilibrium is $W_i = \beta y_{ij}$ where entrepreneur j has exactly the same rank as worker i.

Proof: Since the mobility cost ϵ is arbitrarily small, agents will switch partners until the best allocation of workers to entrepreneurs is achieved. But in this allocation, no worker (or entrepreneur) can be moving to a better partner - as in the proof of Lemma 2 - and since ϵ is still positive, the inside option is preferred to the outside option. Therefore, for all i and j of the same rank, $W_{ij} = \beta y_{ij}$. QED

Both efficient matching and random matching with small mobility costs lead to a situation where heterogenous agents are matched in the "right" way (i.e. the most skilled worker with the most skilled entrepreneur). But given this allocation, the outside options will never be binding because no party can be moving to a better match; thus the "inside" option determines the wages and rates of return to entrepreneurs. Now we can state;

Corollary to Proposition 2: With efficient matching or with random matching as $\epsilon \to 0$, the equilibrium of Proposition 2 is the unique equilibrium of the fully specified economy even when agents are allowed to use non-symmetric strategies.

Proof: Consider a case in which two workers of human capital h_1 and h_2 have different utility levels. If the first worker has higher utility and $h_1 > h_2$, then the second worker can increase his investment to slightly above h_1 (by continuity) and get a better rank thus the same job and the same utility (or arbitrarily close to it). If $h_1 < h_2$, this time he can reduce his investment to slightly above h_1 and the same argument applies. Hence a contradiction and all workers have to obtain the same level of utility and the same applies to all entrepreneurs. Given that the problem we have is strictly convex, this can only be possible, if they all choose the same level of ex ante investment and this gives exactly the same outcome as Proposition 2 as the unique equilibrium even considering non-symmetric outcomes. QED

The intuition of this corollary is straightforward. The wage determination rules of Lemmas 2 and 3 still give concave pay-off functions to each agent which leads to a situation in which they all choose the

same levels of investment ex ante and in this case the result is exactly the same as Proposition 2[20].

Appendix B - Proofs of Propositions 1-6

Proof of Proposition 1: (11) defines optimal decision rules. Substituting from (12) we get

$$\frac{l_t(1+l_t)}{e_t(1-e_t)}=\frac{\alpha}{1-\alpha}\frac{Z_{t-1}}{H_{t-1}} \tag{A2}$$

for all pairs. Balanced growth is only possible when the growth rate of Z_t and that of H_t are equal which implies investment levels

$$l_t=e_t=\{\alpha^{\alpha}(1-\alpha)^{1-\alpha}A(1-\delta)\}^{\frac{1}{\gamma}} \tag{A3}$$

(A3) makes sure that both H_t and Z_t grow at the same rate and output will also grow at this rate g^c. Equation (A2) also implies that if z/h ratio is less than $(1-\alpha)/\alpha$, more entrepreneurial capital than human capital will be accumulated, and vice versa if the ratio is more than $(1-\alpha)/\alpha$. Thus the balanced growth path is globally stable.

Pareto optimality follows from the observation that this allocation maximizes the welfare of generation t given H_{t-1} and Z_{t-1} and that there is no possibility of redistribution welfare across generations, so increasing the investment of current generation to improve the welfare of future generations will not be a Pareto improving action. QED

Proof of Proposition 2: Equation (14) implies that all workers face the same distribution of returns and their problem is concave, thus they will all choose the same level of human capital investment and the same argument applies to entrepreneurs. Imposing this condition we obtain

$$\frac{l_t(1+l_t)}{e_t(1+e_t)}=\frac{\alpha\beta}{(1-\alpha)(1-\beta)}\frac{Z_{t-1}}{H_{t-1}} \tag{A4}$$

for all pairs. Balanced growth is only possible when the growth rate of Z_t and that of H_t are equal which implies

$$l_t=e_t=\{\alpha^{\alpha}(1-\alpha)^{1-\alpha}\beta^{\alpha}(1-\beta)^{1-\alpha}A(1-\delta)\}^{\frac{1}{\gamma}} \tag{A5}$$

and therefore the economy grows at the rate g^d as given in the text. By the same argument as above, if the z/h ratio is different than the one consistent with balanced growth, i.e. $\alpha\beta/(1-\alpha)(1-\beta)$, transitory dynamics

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[20] Efficient matching is not always innocuous. When workers face the risk of being unemployed (as in section 5), higher human capital enables workers to increase their likelihood of being employed and this may lead to an "education race". The intuition is that with unemployment, there is *ex post* heterogeneity and workers will change their *ex ante* investment decisions to influence their likelihoods of ending-up in different groups. If in the model of the section 5 we change the matching technology to efficient matching, an equilibrium fails to exist (details available from the author). These issues are further investigated in Acemoglu (1994). Also similarly, in the presence of non-convexities, small values of ϵ will lead to new effects which we study next.

will bring the economy back to balanced growth.

Both on and off the balanced growth path, if a Social Planner imposes the competitive investment levels, all agents can be made better off. Therefore the dynamic equilibrium is inefficient. The last thing to prove is that pecuniary increasing returns are present in the sense that a small increase in all agents investment will make everyone better-off. Consider such a change. The impact on the welfare of a worker will be proportional to

$$(A6) \qquad (1-\delta)(1-\alpha)(1-\beta)Ah_t^{\alpha}z_t^{-\alpha}dz_t+\{(1-\delta)\alpha\beta Ah_t^{\alpha-1}z_t^{1-\alpha})-l_t^{\gamma}\}dh_t$$

The coefficient of dh_t is equal to zero by the first-order condition and thus the effect on the welfare of the worker will be positive. A similar term applies for the welfare of the entrepreneurs and they too are made better-off. Thus a small increase in the investments of all current agents makes all current agents (and so, all future agents) better-off and the economy is subject to *pecuniary increasing returns*. QED

Proof of Proposition 3: Straightforward maximization of g^d gives $\beta=\alpha$ but by the above argument the equilibrium is still Pareto dominated for all values of β. QED

Proof of Proposition 4: If an entrepreneur chooses the advanced technology when all others are expected to choose the low technology, workers will only invest $l_t(0)$, thus the entrepreneur will make $\{(1-\beta)A_2B_2-k\}H_{t-1}$ and if this is less than $\{(1-\beta)A_1B_1\}H_{t-1}$, there exists an equilibrium in which the advanced technology is not adopted. Conversely, the profit to adoption when all others are anticipated to adopt is $\{(1-\beta)A_2B_2-k\}H_{t-1}$ (since workers now choose $l_t(1)$); if this greater than $\{(1-\beta)A_1B_2\}H_{t-1}$, then there exists an equilibrium in which the new technology is adopted. Since both conditions can be satisfied simultaneously, there exists multiple equilibria. QED

Proof of Proposition 5: First consider the case in which ϵ is positive and τ tends to zero (i.e. all firms choose the low productivity technology). This implies that the worker will have to switch infinitely many partners before finding a firm with the high technology and thus his outside option is not binding and he always receives $\beta A_1 h$. This gives us the same value of $l_t(0)$ and the same condition as in Proposition 4 for investment in the less productive technology to be an equilibrium.

Next consider the case with $\tau\to 1$ and $\epsilon\to 0$. Equation (19) for the wage rate implies that a firm without the more productive technology has to pay exactly the same wage rate as the more productive firms. Thus the return to the low productivity technology is $\{A_1B_2 - \beta A_2B_2\}H_{t-1}$ whereas the return to the advanced technology is as before $\{(1-\beta)A_2B_2-k\}H_{t-1}$. The comparison of these two amounts gives the condition for the new technology to be adopted as $A_2B_2- k > A_1B_2$.

In contrast for the new technology to be adopted in the first best we need;

$$(A7) \qquad \begin{gathered} A_2(1+\{A_2(1-\delta)\}^{\frac{1}{\gamma}})-k-\Delta UC>A_1(1+\{A_1(1-\delta)\}^{\frac{1}{\gamma}}) \\ A_2B_2-k-\Delta UC>A_1B_1 \end{gathered}$$

where $\Delta UC.H_{t-1}$ is the utility cost of higher investment for each worker. It is straightforward to see that (A7) is more restrictive than $A_2B_2-k> A_1B_1$ even when ΔUC is set equal to zero. Therefore, advanced technologies that would not be adopted in the first-best can still be adopted in the decentralized economy if ϵ is small enough. QED

Proof of Proposition 6: Random matching again implies that all workers are facing the same convex maximization problem and they will all choose the same level of investment. Thus the intersections of the two curves denoting the right and left-hand sides of (24) characterize all the equilibria. When no

entrepreneur enters, workers do not invest in human capital at all. If η_0 is greater than $(1-\beta)A(1-\delta)$, an entrepreneur who deviates and decides to enter would not make enough profits to cover her start-up costs given the human capital investments of the workers. However, we also know from (21) that if all firms want to be active, this is infinitely costly for the last firm, thus in the neighborhood of 1, $\mu(E)$ is vertically above the curve that denotes returns to active entrepreneurship. Therefore, if the two curves will intersect at all they must intersect at least two more times. The presence of pecuniary increasing returns can be demonstrated by exactly the same method as in the proof of Proposition 2. To analyze the inefficiency, denote the efficient allocation as (E^*,l^*) such that E^* entrepreneurs become active and each worker chooses investment l^*. It follows from comparing (24) and (25) that if $E=E^*$, then l cannot be equal to l^*, thus the decentralized economy is constrained Pareto inefficient. QED

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